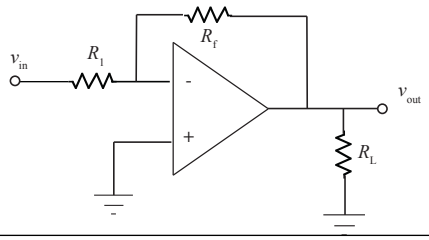
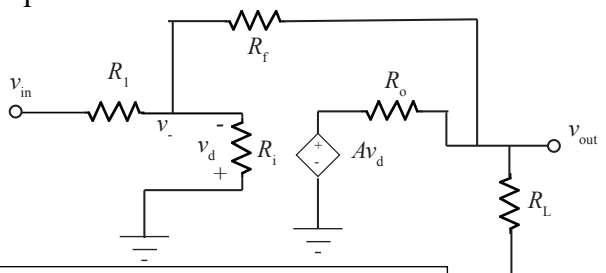
Derivation of Ideal Op Amp Rules



Inverting Amplifier Configuration



Inverting Amplifier with Op Amp equivalent circuit

Nodal Equations:

$$\frac{v_- - v_{in}}{R_1} + \frac{v_- - v_{out}}{R_f} + \frac{v_-}{R_i} = 0 \quad \text{and} \quad \frac{v_{out} - Av_d}{R_o} + \frac{v_{out} - v_-}{R_f} + \frac{v_{out}}{R_L} = 0 \quad , \text{ where } v_d = 0 - v_-$$

Solving, we find:

$$v_- = \frac{v_s / R_1}{\frac{A - R_o / R_f}{R_f + R_o(1 + R_f / R_L)} + \frac{1}{R_1} + \frac{1}{R_f} + \frac{1}{R_i}} \quad \& \; v_{out} = \frac{v_s / R_1}{\frac{R_f + R_o\left(1 + R_f / R_L\right)}{R_o - AR_f}\left(\frac{1}{R_1} + \frac{1}{R_f} + \frac{1}{R_i}\right) - \frac{1}{R_f}}$$

Now, we consider the effects of the Op Amp parameters becoming ideal.

First, for as $R_o \to 0$ (while requiring $R_L \neq 0$) we find:

$$v_- \to \frac{v_s / R_1}{\frac{A}{R_f} + \frac{1}{R_1} + \frac{1}{R_f} + \frac{1}{R_i}} \quad \& \quad v_{out} \to \frac{v_s / R_1}{\frac{-1}{A}\left(\frac{1}{R_1} + \frac{1}{R_f} + \frac{1}{R_i}\right) - \frac{1}{R_f}} \quad \text{for } R_o \to 0,\ R_L \neq 0$$

Next, as $R_i \to \infty$ and $R_f < \infty$, we find:

$$v_- \to \frac{v_s / R_1}{\frac{A}{R_f} + \frac{1}{R_1} + \frac{1}{R_f}} \quad \& \quad v_{out} \to \frac{v_s / R_1}{\frac{-1}{A}\left(\frac{1}{R_1}\right) - \frac{1}{R_f}} \quad \text{for } R_o \to 0, R_i \to \infty\ ,\ R_L \neq 0\ ,\ R_f < \infty$$

And finally, as $A \to \infty$, we have:

$$v_- \to 0 \quad \& \quad v_{out} \to -\frac{R_f}{R_1} v_s \quad \text{for } R_o \to 0, R_i \to \infty,\ A \to \infty\ ,\ R_L \neq 0$$

From these results, we can now formulate the "Ideal Op Amp Rules"

1) The currents into both input terminals are zero, because $R_i \to \infty$
2) Because $v_d = -v_- \to 0$ as long as $A \to \infty$ and $R_f < \infty$, we say that there is a *virtual short* across the input terminals.
3) And … one thing you should NEVER do is to write a nodal equation at the output pin of an ideal Op Amp, since we DON'T KNOW in advance how much current is coming out of this pin. If you write a nodal equation at an output pin, you will die!